Reconstructing Kepler's Reasoning in the Astronomia Nova

A Compact Argument about the Equant, Made Explicit and Checked Step by Step

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Abstract

In chapter 4 of the first part of the *Astronomia nova* Kepler gives, in eight lines and one figure, the argument by which Copernicus charged Ptolemy's equant with making the heavens move unequally: about the centre of its circle the planet passes a lesser arc and a greater arc in equal times. We reconstruct this argument. First the model is made explicit: every point of Kepler's figure with its construction and its astronomical meaning, the one angle that varies, and what is assumed of the lengths. Then the argument is written as steps, each with the premises it uses and the proposition of Euclid that licenses it, none of which Kepler names. The reconstruction is checked by a program: every step is tested over the whole parameter space of the model and all its states, matched against the rule it cites, and tested for the hypotheses it needs. Three things come out that the passage does not say. Kepler argues for the position in which the angle at the equant is right; the central inequality holds in every position of the half revolution. The hypothesis that the equant lies as far above the centre as the eye lies below it, which Kepler states, is not needed for the conclusion. And the argument rests on a triangle that vanishes when the planet is at the apogee or the perigee. A coda treats chapter 2, where Kepler states the equivalence of the eccentric and the concentric with epicycle and leaves the proof to the figure: "Schema loquitur Geometrae". The paper is meant as a model for doing the same with other passages.

KEYWORDS Kepler · Astronomia nova · equant · geometrical reasoning · argument reconstruction · history of astronomy

1 The passage

Chapter 4 of Part I of the *Astronomia nova* [1] introduces the model that Ptolemy uses for the first inequality of the planets. Kepler sets it out in three sentences:

> "Centro B scribatur eccentricus DE, cujus eccentricitas sit BA, ut A sit locus oculi. Acta linea per BA ostendet in D apogaeum in F perigaeum. In hac linea supra B spatium aliud BC extendatur, aequale ipsi BA. Erit C punctum aequantis, punctum nempe, apud quod Planeta aequalibus temporibus conficit aequales angulos, quamvis circulum non circa C sed circa B ordinet."

In Donahue's translation [2]: "About centre B let an eccentric DE be described, with eccentricity BA, A being the place of the observer. The line drawn through AB will indicate the apogee at D and the perigee at F. Upon this line, above B, let another segment BC be extended, equal to BA. C will be the point of the equant, that is, the point about which the planet completes equal angles in equal times, even though the circle is set up around B rather than C."

Then comes the argument. Copernicus, Kepler says, censures this hypothesis because it "offends against physical principles by instituting irregular celestial motions":

> "Eligatur enim E punctum in circulo, quem Planeta corpore peragrat, connectaturque cum CBA : et sit jam DCE rectus, ut et ECF. Cum ergo sint anguli hi aequales, constituti nempe aequalibus temporibus,

> et DCE exterior aequet CBE, CEB interiores : ergo parte CEB ablata, residuus CBE vel DBE minor erit quam DCE. itaque FBE major quam DCE vel FCE. Sed DE arcus metitur DBE angulum, et EF arcus angulum EBF. minor ergo DE quam EF. et transit Planeta per eos aequalibus temporibus."

"Therefore", Kepler concludes, "the entire solid orb is now fast, now slow. This Copernicus rejects as absurd." The passage stands on printed page 73 of the third volume of the *Gesammelte Werke*; in the Famulus collection `kepler-prim` it is paragraphs 3 and 4 of page 71 of the document `kgw-03`, and its figure has the address `fr:kepler-prim/kgw-03/0071/fig01`. Every Latin sentence quoted in this paper was checked against the transcription of that page by the reading tool described in Section 4 (Table 2).

The argument is short, it is about a model, and it is entirely geometrical: no observation and no number enters. That makes it a good first case. What does it take to say precisely what Kepler argues, from what, and by what right?

2 The model made explicit

A reconstruction begins with the model, not with the argument. Kepler's three sentences define a geometry. Written out, it has six points, one circle, one angle that varies and a few lengths (Table 1). We write it in the language astra/1 [3], in which a model is the pure geometry: it names its lengths and angles as symbols and gives them no values.

Table 1 The equant model of Kepler's chapter 4, explicit. Every object has a construction and an astronomical meaning. The model is `fr:astra/kepler-equant`.

	what it is	defined by	astronomical meaning
R	parameter, length	in (10, 1000)	radius of the eccentric circle about B
e	parameter, length	in (0, R / 3)	eccentricity BA: the distance of the eye from the centre
q	parameter, length	in (0, R / 3)	the distance BC of the equant point from the centre
λ_D	parameter, angle	any value	direction of the apogee
T	parameter, time	in (100, 20000)	period of the planet about the equant point
θ_0	parameter, angle	any value	the angle at the equant point at t = 0
θ	kinematic variable	uniform in time, period T	the angle DCE at the equant point, counted from the apogee
A	point	`observer()`	the eye
B	point	`A + polar(e, lamD)`	centre of the eccentric
C	point	`B + polar(q, lamD)`	the equant point
orb	circle	`circle(B, R)`	the eccentric, the circle the planet traverses
D	point	`B + polar(R, lamD)`	apogee
F	point	`B - polar(R, lamD)`	perigee
E	point	`seen_from(C, orb, lamD + theta)`	the planet: where the line from C at the angle θ meets the circle
λ	observable	`direction(A, E)`	longitude of the planet as seen from A

	what it is	defined by	astronomical meaning
atC	observable	`angle(D, C, E)`	the angle DCE at the equant point
atB	observable	`angle(D, B, E)`	the angle DBE at the centre: the arc DE the planet has traversed

Three decisions were taken in writing the model, and each is a decision about what Kepler says.

Whose model it is. Kepler presents the hypothesis as Ptolemy's ("PTOLEMAEVS … operosiori utitur hypothesi"). The model is therefore attributed to Ptolemy; Kepler's paragraph is the witness, the text whose use of the model is analysed; the description is the author's. These are three different roles.

What varies. The planet E is placed where the line from C at the angle θ meets the circle about B. The angle θ at the equant point is the one kinematic variable. That the planet "completes equal angles in equal times" about C is the law of this variable: θ grows uniformly with time. Nothing else in the model moves by itself.

What is a hypothesis. Kepler makes BC equal to BA. In the model the distance BC is a parameter of its own, q, beside the eccentricity e. The equality q = e is then a hypothesis of the argument, not a part of the geometry. This costs nothing and allows a question to be asked later: does the argument need it?

Figure 1 shows the model drawn from its text in the position Kepler chooses.

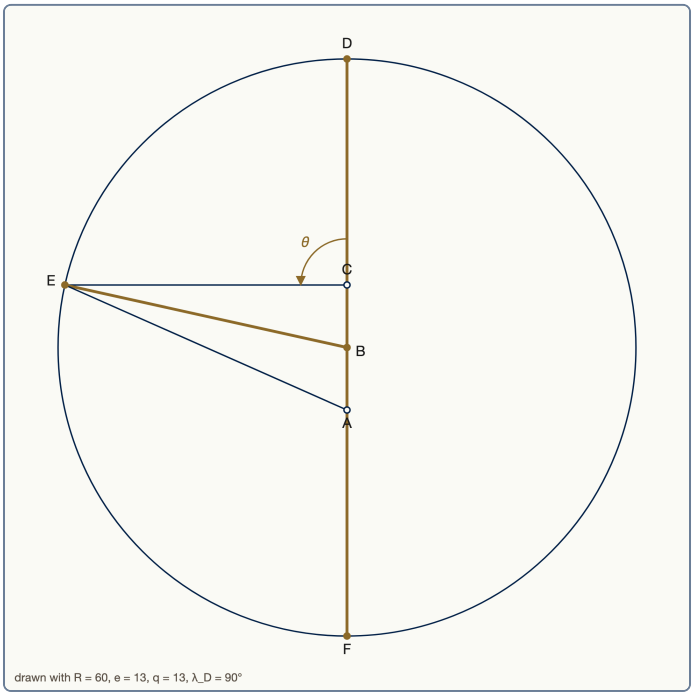
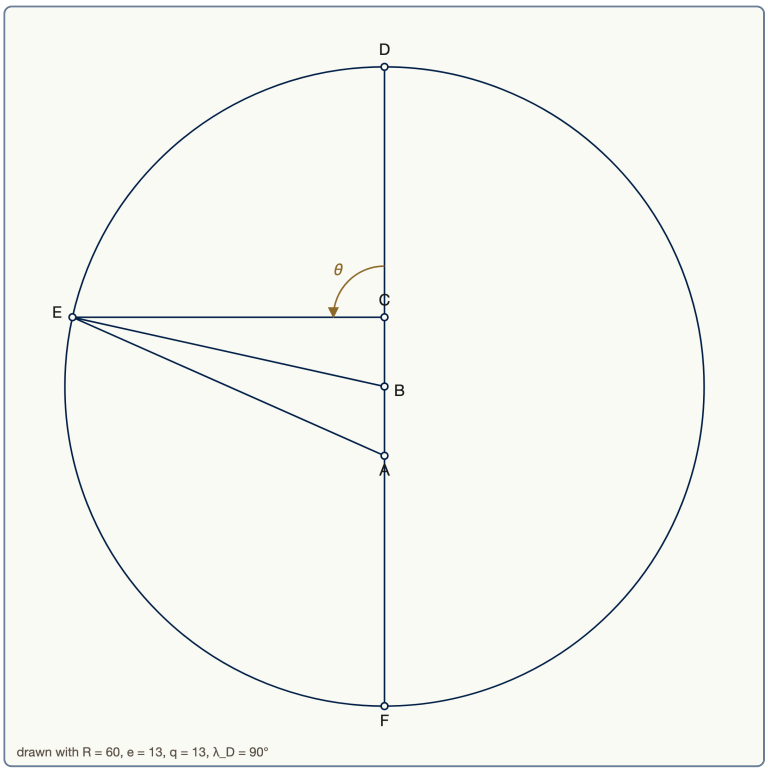


Figure 1 The equant model in the position of Kepler's argument: the angle DCE at the equant point C is right. A is the eye, B the centre of the circle, D the apogee, F the perigee, E the planet. In gold the lines of the first step: the triangle CBE with its exterior angle at C. The lengths are drawing values. Kepler's printed figure has the same points and one more, J, which belongs to a later argument.

The same figure in motion, drawn from the model: the angle θ at the equant point C runs through a revolution and carries the planet E with it. The lengths are drawing values.



3 The argument reconstructed

Kepler's paragraph contains six assertions and two words of inference, "ergo" and "itaque". A reconstruction makes every inference a step with three parts: the proposition, the premises it is drawn from, and the rule that licenses the drawing. Figure 2 is the result. It is printed from the same document that drives the interactive form of the argument.

About the centre of its circle the planet moves unequally: the arc DE is less than the arc EF, and both are passed in equal times

GIVEN

H1 the equant point lies as far above the centre as the eye lies below it: BC = BA
q = e

ARGUMENT

1 **the angle DBE at the centre is less than the angle DCE at the equant point** ✓
∠DBE < ∠DCE from 1.4

1.1 DCE is the exterior angle of the triangle CBE, equal to the interior angles CBE and CEB — *Elements I.32* ✓
∠ECD = ∠CEB + ∠EBC

1.2 the part CEB taken away, the remainder CBE is less than DCE — *common notion 5* ✓
∠EBC < ∠ECD from 1.1

1.3 CBE or DBE: C and D lie from B on one ray — *construction* ∘
∠EBC = ∠DBE from constr. C, constr. D, constr. E

1.4 ∠DBE < ∠DCE — *common notion 1* ✓
from 1.2, 1.3

2 BE stands on the apsidal line at B: DBE and EBF make two right angles — *Elements I.13* ✓
∠DBE + ∠EBF = 180°

3 CE stands on the apsidal line at C: DCE and ECF make two right angles — *Elements I.13* ✓
∠DCE + ∠ECF = 180°

4 **so the angle FBE is greater than the angle FCE** ✓
∠ECF < ∠EBF from 4.1

4.1 ∠ECF < ∠EBF — *algebra* ≈
from 1, 2, 3

5 Suppose: let DCE be a right angle, and so ECF: two quarters of the period ·
∠DCE a right angle

6 **the arc DE, which measures the angle DBE, is less than the arc EF, which measures the angle EBF** ✓
∠DBE < ∠EBF from 6.2

6.1 the angles at C are equal: they are made in equal times — *algebra* ≈
∠DCE = ∠ECF from 5, 3

6.2 DBE < DCE = ECF < EBF — *algebra* ≈
∠DBE < ∠EBF from 1, 4, 6.1

THEREFORE

∴ **in equal times the planet passes the lesser arc DE and the greater arc EF: the orb is now slow, now fast** ✓
∠DBE < ∠EBF from 6

Figure 2 Kepler's argument about the equant, reconstructed. Indented blocks are sub-arguments: the claim stands first, the steps that establish it below. At the right the rule of each step; ✓ an instance of the rule named, ◦ read off the construction, ≈ by calculation. "from" names the premises. H1 is Kepler's hypothesis BC = BA.

The argument has three parts and a supposition.

The angle at the centre lags behind the angle at the equant (1). DCE is the exterior angle of the triangle CBE. Euclid's proposition I.32 says that "the exterior angle is equal to the two interior and opposite angles" [4, vol. I, p. 316]: DCE = CEB + CBE. Kepler: "DCE exterior aequet CBE, CEB interiores". Take the part CEB away and the remainder CBE is less than DCE — "the whole is greater than the part", Common Notion 5. CBE and DBE are one angle, since C and D lie from B on one ray; Kepler writes "CBE vel DBE". So DBE is less than DCE.

The supplement leads (4). DBE and EBF together make two right angles, and so do DCE and ECF (I.13: "if a straight line set up on a straight line make angles, it will make either two right angles or angles equal to two right angles"). Since DBE is less than DCE, EBF is greater than ECF. Kepler: "itaque FBE major quam DCE vel FCE". His "itaque" carries two applications of I.13 and a subtraction.

The supposition (5). "Et sit jam DCE rectus, ut et ECF." Kepler fixes the position: the angle at the equant is right, so that the planet has used a quarter of its period from D to E and will use another quarter from E to F. From here on the argument speaks of this position only.

Unequal arcs in equal times (6). In this position DCE equals ECF. So DBE is less than DCE, which equals ECF, which is less than EBF. The arc DE measures the angle DBE at the centre and the arc EF the angle EBF: the arc DE is less than the arc EF, "et transit Planeta per eos aequalibus temporibus".

Kepler names none of the three propositions. A reader of 1609 did not need them named. A reconstruction does, because the name of the rule is what makes a step checkable.

4 What the checks say

The reconstruction is a text in a formal language (Listing 1), and a program asks four questions of it [5]. Is every premise there? Is every proposition true — in the model, for every admissible value of its lengths and every position of the planet? Is every step an instance of the rule it names, with the hypotheses of that rule among the cited premises and with what the rule takes from the figure actually holding? And does the conclusion rest only on what is cited?

Listing 1 The reconstruction as an astra/1 derivation (`fr:astra/kepler-equant-unequal-motion`), comments removed. `show … qed` is a sub-argument, `suppose` restricts what follows to the positions in which its condition holds.

```
derivation kepler-equant-unequal-motion in kepler-equant "About the centre of its
      circle the planet moves unequally: the arc DE is less than the arc EF, and
      both are passed in equal times"
witness fr:kepler-prim/kgw-03/0071/p05 "Astronomia nova ch. 4"
witness fr:kepler-sec/Kepler1992/0147/p02 "Donahue's translation"
author "Gerd Graßhoff"
given h1: equal(q, e) "the equant point lies as far above the centre as the eye
      lies below it: BC = BA"
```

```
show lag: less(angle(D, B, E), angle(D, C, E)) "the angle DBE at the centre is
      less than the angle DCE at the equant point"
  derive k1: equal(angle(E, C, D), angle(C, E, B) + angle(E, B, C)) by Elements
        I.32 "DCE is the exterior angle of the triangle CBE, equal to the interior
        angles CBE and CEB"
  derive k2: less(angle(E, B, C), angle(E, C, D)) from k1 by common notion 5 "the
        part CEB taken away, the remainder CBE is less than DCE"
  derive k3: equal(angle(E, B, C), angle(D, B, E)) from def C, def D, def E by
        construction "CBE or DBE: C and D lie from B on one ray"
  derive k4: less(angle(D, B, E), angle(D, C, E)) from k2, k3 by common notion 1
qed from k4
derive k5: equal(angle(D, B, E) + angle(E, B, F), 180°) by Elements I.13 "BE
      stands on the apsidal line at B: DBE and EBF make two right angles"
derive k6: equal(angle(D, C, E) + angle(E, C, F), 180°) by Elements I.13 "CE
      stands on the apsidal line at C: DCE and ECF make two right angles"
show lead: less(angle(E, C, F), angle(E, B, F)) "so the angle FBE is greater than
      the angle FCE"
  derive k7: less(angle(E, C, F), angle(E, B, F)) from lag, k5, k6 by algebra
qed from k7
suppose quadrant: right_angle(D, C, E) "let DCE be a right angle, and so ECF: two
      quarters of the period"
show unequal: less(angle(D, B, E), angle(E, B, F)) "the arc DE, which measures the
      angle DBE, is less than the arc EF, which measures the angle EBF"
  derive k8: equal(angle(D, C, E), angle(E, C, F)) from quadrant, k6 by algebra
        "the angles at C are equal: they are made in equal times"
  derive k9: less(angle(D, B, E), angle(E, B, F)) from lag, lead, k8 by algebra
        "DBE < DCE = ECF < EBF"
qed from k9
conclude less(angle(D, B, E), angle(E, B, F)) from unequal "in equal times the
      planet passes the lesser arc DE and the greater arc EF: the orb is now slow,
      now fast"
```

The answer to all four is yes (Listing 2). Of the nine steps, five are instances of I.32, I.13 and the two Common Notions, one is read off the construction, three are calculations with the cited equalities and inequalities. The test runs over 48 sampled sets of lengths and 72 positions of the planet.

Listing 2 The report of the checks.

```
$ famulus-astra derive kepler-equant-unequal-motion
kepler-equant-unequal-motion: derivation in kepler-equant, level conclusive (5/5)
  ✓ well-formed
  ✓ grounded
  ✓ true
  ✓ justified
  ✓ conclusive
  given     h1: equal(q, e)
  ✓ lag: less(angle(D, B, E), angle(D, C, E)) from k4 — justified (it is k4)
    ✓ k1: equal(angle(E, C, D), angle(C, E, B) + angle(E, B, C)) by Elements I.32
          — justified (configuration conditions fail there: noncollinear(E, B, C))
      degenerate at theta = 0°, theta = 180°
    ✓ k2: less(angle(E, B, C), angle(E, C, D)) from k1 by common notion 5 —
          justified
    ✓ k3: equal(angle(E, B, C), angle(D, B, E)) from def C, def D, def E by
          construction — read off the construction
    ✓ k4: less(angle(D, B, E), angle(D, C, E)) from k2, k3 by common notion 1 —
```

```
        justified
✓ k5: equal(angle(D, B, E) + angle(E, B, F), 180°) by Elements I.13 — justified
✓ k6: equal(angle(D, C, E) + angle(E, C, F), 180°) by Elements I.13 — justified
✓ lead: less(angle(E, C, F), angle(E, B, F)) from k7 — justified (it is k7)
  ✓ k7: less(angle(E, C, F), angle(E, B, F)) from lag, k5, k6 by algebra — by
        calculation
· quadrant: right_angle(D, C, E) — supposed
✓ unequal: less(angle(D, B, E), angle(E, B, F)) from k9 — justified (it is k9)
  ✓ k8: equal(angle(D, C, E), angle(E, C, F)) from quadrant, k6 by algebra — by
        calculation
  ✓ k9: less(angle(D, B, E), angle(E, B, F)) from lag, lead, k8 by algebra — by
        calculation
✓ conclusion: less(angle(D, B, E), angle(E, B, F)) from unequal — justified (it
      is unequal)
note: given h1 (equal(q, e)) is not needed for the conclusion
note: degenerate states, in which a step's rule does not apply, a term is
      undefined or an inequality becomes an equality: theta = 0°; theta = 180°.
      The derivation does not cover them; the conclusion is true there as well
      (numerically)
```

The quotations are checked separately. Each sentence of Kepler that the reconstruction rests on is an item of a reading file: the quotation is searched in the transcription of the paragraph, and the assertion is tied to the step that renders it (Table 2).

Table 2 The sentences of Kepler used in this paper, with the paragraph where each stands and the step or proposition it is checked as. ✓ confirmed; — a statement that is not a step of the geometry. Items 8 to 11 belong to chapter 2 (Section 6).

#	place	quoted	checked as	verdict
1	`0071/p04`	Erit C punctum aequantis, punctum nempe, apud quod Planeta aequalibus temporibus conficit aequa…	proposition `p1`	✓
2	`0071/p04`	spatium aliud BC extendatur, aequale ipsi BA	not formal –	—
3	`0071/p05`	DCE exterior aequet CBE, CEB interiores	step `k1`	✓
4	`0071/p05`	ergo parte CEB ablata, residuus CBE vel DBE minor erit quam DCE	step `lag`	✓
5	`0071/p05`	itaque FBE major quam DCE vel FCE	step `lead`	✓
6	`0071/p05`	minor ergo DE quam EF. et transit Planeta per eos aequalibus temporibus	step `unequal`	✓
7	`0071/p05`	Quod COPERNICVS ut absurdum rejicit	not formal –	—
8	`0064/p04`	eccentricus paria facit cum epicyclo in concentrico	step `conclusion`	✓
9	`0064/p04`	hic semidiameter epicycli aequet illic eccentricitatem, semidiametri vero illic eccentri et hic…	not formal –	—

#	place	quoted	checked as	verdict
10	`0064/p05`	Dico, distantias AC	step `same`	✓
11	`0065/p02`	Nec verbis opus est. Schema loquitur Geometrae.	not formal –	—

5 What the reconstruction shows

Three things come out of the checks that Kepler's paragraph does not say.

The inequality holds in every position, not only at the right angle. Kepler introduces the right angle before he states the exterior angle. In the reconstruction the supposition stands where it is first needed, after the two inequalities, and the checks confirm that it can stand there: the angle at the centre is less than the angle at the equant in every position of the planet between apogee and perigee, and its supplement is greater (Figure 3). The right angle is needed for one thing only: to make the two angles at C equal, so that two arcs can be compared that are passed in equal times. What Kepler shows for two quarters of the period is a property of the model throughout: measured from the apogee, the planet has always turned less about the centre of its circle than about the equant point.

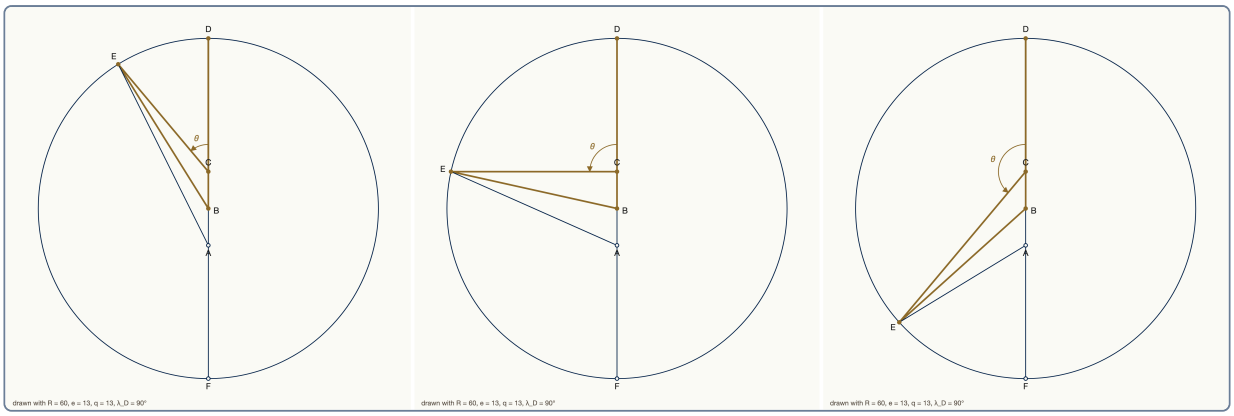


Figure 3 The first inequality in three positions: the angle DCE at the equant is 40°, 90° and 140°. In each the angle DBE at the centre is the smaller one. Kepler argues the middle position.

The bisection is not needed. Kepler states that BC is equal to BA, and this is what makes the model Ptolemy's. For every hypothesis the program repeats the test with that hypothesis lifted, here with the equant at any distance above the centre. The conclusion still holds, and so does every step. The report notes: "given h1 is not needed for the conclusion". The argument against the equant does not turn on where the equant point lies, only on its not being the centre. That is worth knowing when one reads on: the next pages of the chapter compare Ptolemy's model with Copernicus's replacement, and there the bisection matters.

At the apsides the triangle vanishes. I.32 speaks of a triangle. When the planet is at D or at F, the points C, B and E lie on one line and there is none. The program evaluates, for every step that is an instance of a rule, what the rule takes from the figure — here that E, B, C form a triangle and that C lies between B and D — in every sampled position, and reports the positions in which it fails: $\theta = 0°$ and $\theta = 180°$. In these two positions the strict inequality becomes an equality, as it must. Kepler's argument does not touch them, since it supposes the right angle; the general statement of the previous paragraph has to exclude them.

None of this corrects Kepler. It states what his argument establishes, how far beyond his chosen position it reaches, and which of the things he says he uses.

6 Coda: "the figure speaks to the geometer"

Two chapters earlier Kepler treats the simplest equivalence of all: an eccentric circle produces the same as an epicycle on a concentric, "ubi eccentricus paria facit cum epicyclo in concentrico", provided that the line of apsides in the one and the line from the centre of the epicycle to the planet in the other stay parallel, that "hic semidiameter epicycli aequet illic eccentricitatem", and that the radii of the eccentric and of the concentric are equal. He draws two figures (Figure 4), states what is to be shown — "Dico, distantias AC, αγ, aequales esse" and the angles at the eye likewise — and then stops:

> "Hoc inquam PTOLEMAEVS demonstravit lib. III. Nec verbis opus est. Schema loquitur Geometrae."

Ptolemy has demonstrated this; no words are needed; the figure speaks to the geometer.

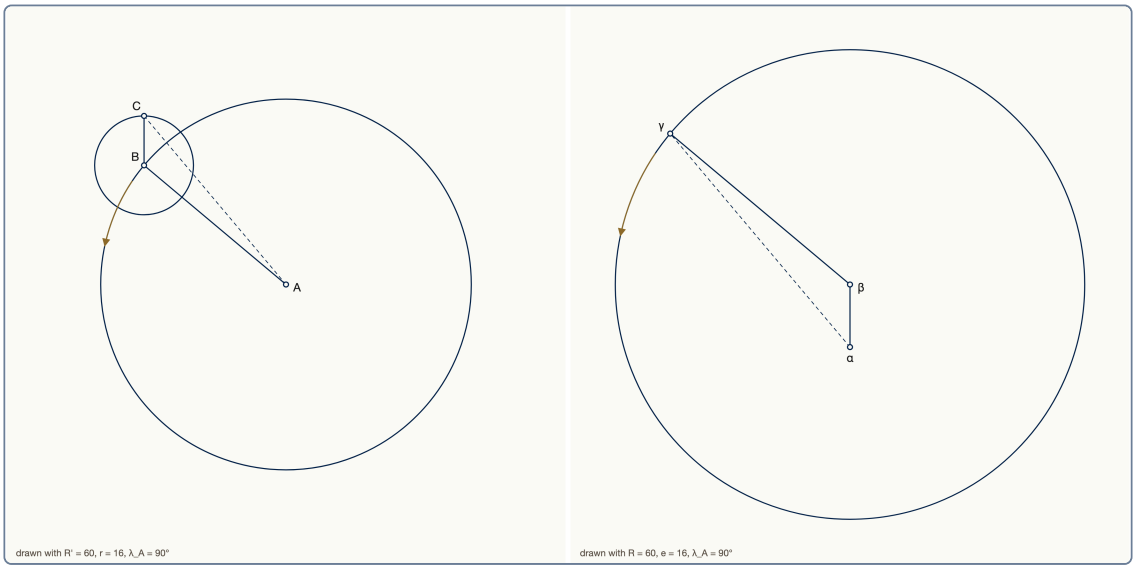


Figure 4 Kepler's two figures for chapter 2, drawn from the two models with his letters: left the concentric with epicycle (the eye A, the centre B of the epicycle, the planet C), right the eccentric (the eye α, the centre β, the planet γ). The lengths are drawing values.

What the figure says can be written down. The two models are those of Ptolemy's solar theory, and their equivalence has been reconstructed elsewhere from Swerdlow's exposition of it [6]. Laid on one another with one eye, the two figures form a parallelogram: the radius of the eccentric to the planet and the radius of the concentric to the centre of the epicycle are equal and parallel, so the lines that join their ends are equal and parallel (Elements I.33), and the planet of the one model is the planet of the other. Figure 5 gives the argument at its coarsest depth.

The two forms place the sun at the same point when R' = R and r = e

GIVEN

H1 the deferent has the radius of the eccentric
R′ = R

H2 the epicycle has the eccentricity as its radius
r = e

ARGUMENT

1 **the sun of the eccentric form and the sun of the epicyclic form are one point** *cases* ✓
S = S′

1.1 S lies on the eccentric *construction* ∘
ES = R from constr. S

1.2 C lies on the deferent *construction* ∘
OC = R′ from constr. C

1.3 OE is the eccentricity *construction* ∘
OE = e from constr. E

1.4 S' lies on the epicycle — *construction* ∘
CS′ = r from constr. S′

1.5 **Case: off the apsidal line, OESC is a parallelogram** ✓
when OES a triangle from 1.5.7

1.5.1 **ES and OC are parallel** ✓
ES ∥ OC from 1.5.1.2

1.5.1.1 both angles are the mean anomaly κ̄, counted from the apsidal line — *construction* ∘
∠AES = ∠A′OC from constr. A, constr. S, constr. A′, constr. C

1.5.1.2 the apsidal line falls on ES and OC and makes equal angles with them on the same side — *Elements I.28* ✓
ES ∥ OC from 1.5.1.1

1.5.2 ES = R = R' = OC — *common notion 1* ✓
ES = OC from 1.1, 1.2, H1

1.5.3 **OE and CS are parallel: OESC is a parallelogram** ✓
OE ∥ CS from 1.5.3.1

1.5.3.1 the lines joining the ends of the equal and parallel ES and OC — *Elements I.33* ✓
OE ∥ CS from 1.5.1, 1.5.2

1.5.4 and they are equal — *Elements I.33* ✓
OE = CS from 1.5.1, 1.5.2

1.5.5 CS = OE = e = r = CS' — *common notion 1* ✓
CS = CS′ from 1.5.4, 1.3, H2, 1.4

1.5.6 the radius of the epicycle keeps the direction of the apsidal line — *construction* ∘
CS′ ∥ OE from constr. S′, constr. E

1.5.7 S and S' lie from C along the parallel to OE, the same way, at the same distance — *lemma same ray* ✓
S = S′ from 1.5.3, 1.5.6, 1.5.5

1.6 **Case: at the apogee the four points lie on the apsidal line, E between O and S** ✓
when O, E, S on one line; E between O and S from 1.6.7

1.6.1 OS = OE + ES — *lemma segment addition* ✓

1.6.2 OS' = OC + CS' — *lemma segment addition* ✓
OS′ = OC + CS′

1.6.3 e + R = R' + r — *algebra* ≈
OS = OS′ from 1.6.1, 1.6.2, 1.1, 1.2, 1.3, 1.4, H1, H2

1.6.4 S lies on the apsidal line — *lemma one line* ✓
OS ∥ OE from the case

1.6.5 and so does S' — *construction* ∘
O, E, S′ on one line from constr. C, constr. S′, constr. E

1.6.6 OS′ ∥ OE — *lemma one line* ✓
from 1.6.5

1.6.7 S and S' lie from O along the apsidal line, the same way, at the same distance — *lemma same ray* ✓
S = S′ from 1.6.4, 1.6.6, 1.6.3

1.7 **Case: at the perigee the four points lie on the apsidal line, O between E and S** ✓
when O, E, S on one line; O between E and S from 1.7.7

1.7.1 ES = EO + OS — *lemma segment addition* ✓

1.7.2 CO = CS' + S'O — *lemma segment addition* ✓
CO = CS′ + S′O

1.7.3 R − e = R' − r — *algebra* ≈
OS = OS′ from 1.7.1, 1.7.2, 1.1, 1.2, 1.3, 1.4, H1, H2

1.7.4 S lies on the apsidal line — *lemma one line* ✓
OS ∥ OE from the case

1.7.5 and so does S' — *construction* ∘
O, E, S′ on one line from constr. C, constr. S′, constr. E

1.7.6 OS′ ∥ OE — *lemma one line* ✓
from 1.7.5

1.7.7 S and S' lie from O along the apsidal line, the same way, at the same distance — *lemma same ray* ✓
S = S′ from 1.7.4, 1.7.6, 1.7.3

THEREFORE

∴ **in every state the two forms have the sun at one point** ✓
S = S′ from 1

Figure 5 The equivalence of the eccentric and the concentric with epicycle, at the depth *overview*: the claim, that it is argued by cases, and the conclusion. The full argument has twenty-six steps in three cases.

Two things in it are of interest here. Kepler's three conditions are exactly what the reconstruction needs: the parallel lines are part of the construction of the epicyclic model, and the two equalities of lengths are its two hypotheses; the test with each lifted finds that the conclusion needs both. And the figure does not speak in every position. When the planet is at the apogee or at the perigee, the parallelogram collapses into the line of apsides, and the argument needs two further cases, in which lengths along one line are added and subtracted. A figure drawn in a general position cannot say that.

7 How to reconstruct an argument

The procedure used here is the same for any geometrical passage about a model.

1. **Fix the passage.** Take the text from an edition with addressable paragraphs; quote it; have the quotations checked against the transcription.
2. **Write the model.** Every point, line and circle of the figure gets a construction from what precedes it, and a meaning. Lengths and angles are symbols. Decide what varies. Say whose model it is.
3. **Separate hypotheses from geometry.** What the author assumes of the lengths, and could have assumed otherwise, is a hypothesis of the argument.
4. **Write the steps.** One proposition per step, in the author's order, in the author's words as its gloss. For every "therefore" name the premises and find the proposition that licenses it. Where the author fixes a position, write a supposition, and place it where it is first needed.
5. **Run the checks.** Grounded, true, justified, conclusive.
6. **Read the report as a finding about the text.** Which steps are instances of propositions, which are read off the figure, which hypotheses are needed, which positions are not covered.

The result is a text that can be read at three depths: the claim and its main parts; every step with its rule; and every step with the binding of its rule, what the rule takes from the figure, and where it fails.

8 Limits

The checks test truth on samples of the parameter space and of the positions; they do not prove. A step counts as an instance of a proposition when it matches a schema of that proposition; the schema states one case, and the wording of the proposition was compared with Heath's translation [4], not with the Greek. Three of the nine steps are calculations that the program confirms numerically without replaying them. Arcs are compared through the angles at the centre that measure them, as Kepler does; the length of an arc is not computed.

The reconstruction covers eight lines of chapter 4. The same chapter goes on to Copernicus's replacement of the equant by a second epicycle, to the remark that the planet's path is then not a circle, and to a numerical comparison of the two models for Mars; these are arguments of the same kind and are not treated here. The physical reading that Kepler attaches to the equant in the following paragraph is not a geometrical argument and lies outside this method.

The Latin is the transcription of the collection, produced by optical character recognition and checked here only for the quoted sentences. The volume's record in the collection names its editors and publisher and no year; none is given in the bibliography.

9 Conclusion

Eight lines of Kepler became a model with six points, one hypothesis, one supposition and nine steps under three propositions of Euclid and two Common Notions. Checked, the argument is sound as far as it goes, and it goes further than Kepler takes it: the inequality between the angle at the centre and the angle at the equant holds in every position, and it does not depend on the bisection of the eccentricity. Where Kepler leaves an argument to the figure, as in chapter 2, the reconstruction says what the figure says and where it falls silent.

Availability

The model `fr:astra/kepler-equant`, the derivation `fr:astra/kepler-equant-unequal-motion`, the figure texts and the reading file `readings/kepler-an-ch4.yaml` are in the Famulus workspace `famulus-astronomica` (`examples/kepler/`). The interactive form of the argument is the element `<astra-argument>`; the page version of this issue shows it as Figure 2. The language and the checks are described in [3] and [5]. Kepler is cited by the pages of the Famulus collection `kepler-prim`, document

`kgw-03` (page n of the collection is printed page n + 2); Donahue's translation by the collection `kepler-sec`, document `Kepler1992`.

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Bibliography

[1] J. Kepler, *Astronomia nova*, vol. 3. in Gesammelte Werke, ed. Walther von Dyck, Max Caspar and Franz Hammer, vol. 3. München: C. H. Beck.

[2] J. Kepler, *New Astronomy*. Cambridge University Press, 1992.

[3] G. Graßhoff, "astra/1. A Language for Historical Astronomical Models and the Geometrical Reasoning about Them," 2026.

[4] Euclid, *The Thirteen Books of Euclid's Elements. Translated from the Text of Heiberg with Introduction and Commentary*. Cambridge: Cambridge University Press, 1908.

[5] G. Graßhoff, "Geometrical Reasoning in the History of Astronomy. Fundamentals," 2026.

[6] G. Graßhoff, "Two Geometrical Arguments Reconstructed. Euler on the Tide-Raising Force, and the Equivalence of Eccentric and Epicycle," 2026.