Reading Five Pages Deeply

Swerdlow on Ptolemy's Solar Model, Checked Assertion by Assertion, with Two Misprints Found

Gerd Graßhoff iD
Humboldt-Universität zu Berlin, Deutsches Museum
gerd.grasshoff@hu-berlin.de

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Abstract

Five pages of Noel Swerdlow's posthumous *The Renaissance of Astronomy* (2026) set out Ptolemy's solar model: the length of the year, the eccentric and the epicyclic form and their equivalence, the equation of centre, Ptolemy's derivation of the parameters from the seasons, and the later values of al-Battānī, az-Zarqāl and the Alfonsine Tables. We read these pages deeply. Every assertion is quoted and the quotation verified against the transcription. Every assertion is then checked as what it is: a property of the construction as a proposition of a model written in the language astra/1, a statement that something follows as a step of a derivation with its rule, a number as sexagesimal arithmetic or as a quantity computed from the model, parameters found from observations as a determination through the model. Every result is shown with the crop of the page it checks. Of 45 assertions, 35 are confirmed, one is a historical judgement that arithmetic cannot reach, and nine differ from recomputation. Seven of the nine concern figures that Swerdlow reports from his sources and are facts about those sources: Ptolemy's rounded arcs of mean motion, al-Battānī's maximum equation, and Ptolemy's eccentricity and apogee, which follow from his rounded arcs and not from the day counts of the seasons. Two concern the page itself and are misprints: the sign rule for the equation of centre names the same condition in both of its clauses, and the advance of al-Battānī's apogee is printed as 16;37° where the two longitudes on the page differ by 16;47°. The derivation of the equivalence, written out and checked step by step, shows further that the parallelogram argument does not apply at the apsides. The reading is a record that runs, every item of which points back to its paragraph.

KEYWORDS history of astronomy · Ptolemy · close reading · verification · sexagesimal arithmetic · scholarly infrastructure

1 Five pages

Noel Swerdlow's *The Renaissance of Astronomy* [1] is a posthumous book of more than a thousand pages on Regiomontanus's *Epitome of the Almagest* and what followed from it. Its first chapter walks through the *Epitome* book by book. Five pages of that chapter concern the sun: page 17 on the length of the tropical year; pages 19 and 20 on the eccentric and the epicyclic form of Ptolemy's solar model and the parallelogram that shows them equivalent; pages 22 and 23 on the complete model, its equation of centre, Ptolemy's derivation of the parameters from the seasons, and the values that al-Battānī, az-Zarqāl and the Alfonsine Tables found later. In the Famulus collection `kepler-sec` these are scan pages 47, 49, 50, 52 and 53 of the document `Swerdlow2026`; every paragraph has an address, `fr:kepler-sec/Swerdlow2026/0052/p05` for instance (the register counts the running head of a page as its first block, so the fourth paragraph of this page is `p05`).

These pages are dense. They contain three figures, two formulae, some sixty numbers — many of them sexagesimal, many of them derived from others on the same page — a construction described in prose, an equivalence claimed between two models, and a sign rule. A reader is asked to believe all of it. Most of it is right. This paper asks what it takes to know that, and what is found when one checks.

The method we call a *deep reading*: every assertion on the pages is isolated, quoted, formalised and checked, and the check is recorded so that another reader can run it again. It was made possible by two pieces of infrastructure: the Famulus reference scheme, which gives every paragraph and figure an address and a transcription, and the language astra/1 [2], in which Ptolemy's solar model, its values and the reasoning

about it are written as checkable texts. The deep reading is the use of both on one passage. The model read here is Ptolemy's; the text read is Swerdlow's exposition of it.

2 Method

A deep reading is a list of *items*, kept as a YAML file (`readings/swerdlow-solar.yaml` in the famulus-astronomica workspace). Each item has:

- a **locus**, the address of the paragraph (`fr:kepler-sec/Swerdlow2026/0052/p06`);
- a **quotation**, the assertion as printed. The tool verifies that the quotation occurs in the paragraph's transcription (the page XML of the collection), after reducing the transcription's LaTeX markup to plain symbols and ignoring whitespace. A quotation that is not found fails the run: the reading is tied to the text, not to memory;
- a **kind**: definition, construction, formula, sign rule, value, computation, derivation, or historical claim;
- a **formalisation**, which says what kind of assertion it is. Arithmetic in sexagesimal notation (`82;17 - 65;30`). A proposition stated in a model (`proposition solar-eccentric: p1`), which is confirmed when it holds over the model's whole parameter space and all its states. A step of a derivation (`step solar-equivalence-equal-radii: d6`), which is confirmed when the step is true and is licensed by the rule it names. A quantity computed or determined in a derivation (`computation solar-parameters-from-seasons: arcs.e`). A sign rule read off a model under a parametrisation (`sign solar-eccentric[solar-eccentric.ptolemy]: c`). Or a value solved from a model: the value of one observable in the state where another has a given value;
- the **printed result** and the number of sexagesimal places to which it is compared; the tolerance is half a unit of the last printed place;
- an **attribution**: whether the assertion is Swerdlow's own statement or arithmetic (`print`) or a figure he reports from a source (`reported`). A discrepancy in the first is about the page; in the second, about the source;
- the **evidence**: a crop of the page image (fractions of the page's width and height), produced by the tool.

`famulus-astra reading readings/swerdlow-solar.yaml` runs the list and writes a report: for each item the verdict *confirmed*, *discrepancy* (with the recomputed value and the difference), *not checkable* (with the reason) or *quotation not found*, and the crop. The report in this paper is that output, reformatted for the page; nothing in the tables below was typed in by hand.

The geometrical assertions are checked against astra/1 texts written from the same pages [2]. These keep four things apart that the pages present together. The *models* are the pure geometry of the eccentric and of the epicyclic form, without values. The *parametrisations* are Ptolemy's values and al-Battānī's, each value with its paragraph. The *superposition* places the two forms in one plane, as Swerdlow's Figs. 1.2 and 1.3 do. The *derivations* are the reasoning: the equivalence of the two forms under the hypotheses R′ = R and r = e, their equivalence in direction under R′/r = R/e, the equation of centre, and the determination of the parameters from the seasons. A property of the construction — "r is always parallel to the apsidal line AB" — is confirmed as a proposition of a model, for every parameter value. A statement of the argument — "complete parallelogram OESC" — is confirmed as a step of a derivation: here the step that OE and CS are parallel, by Elements I.33 (Figure 1).

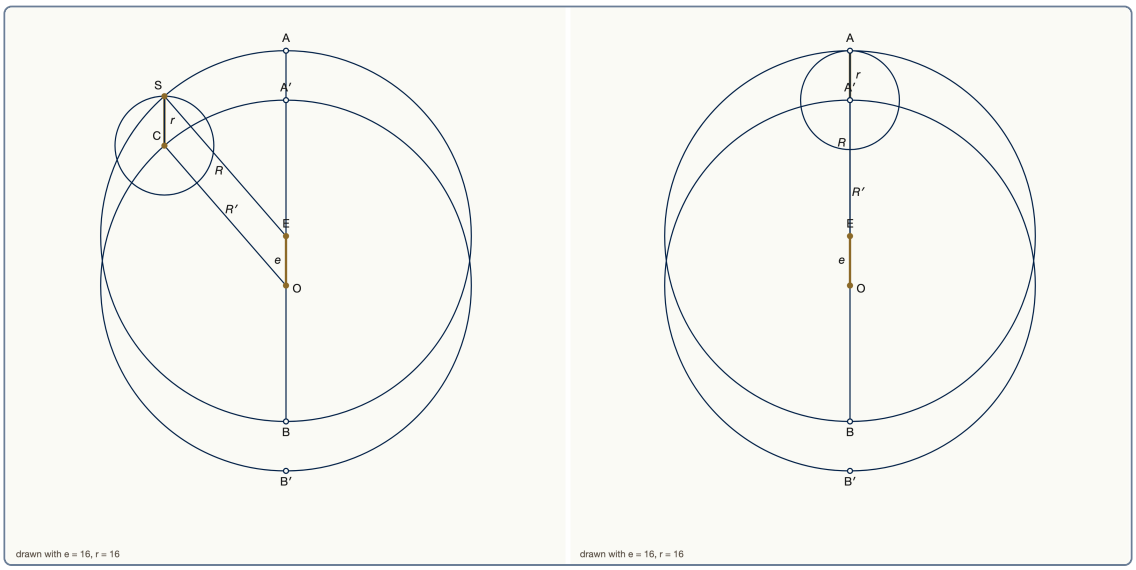
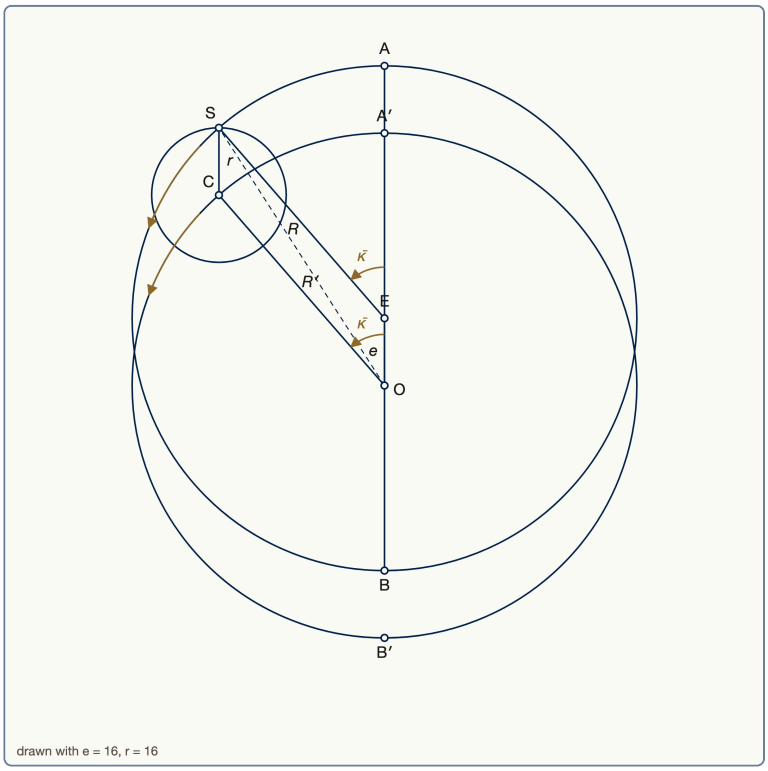


Figure 1 The scene of the derivation of the equivalence. Left, at $\bar{\kappa} = 41°$, the state Swerdlow's Fig. 1.2 shows: OE and CS, in gold, are parallel because ES and OC are equal and parallel (Elements I.33); OESC is the parallelogram of page 19. Right, the same at $\bar{\kappa} = 0°$: the four points lie on the apsidal line and there is no parallelogram.

Swerdlow's Fig. 1.2 in motion, drawn from the two forms of the model superposed: the mean anomaly $\bar{\kappa}$ runs through a revolution, and the slider below the figure fades from the eccentric form to the epicyclic. The eccentricity is exaggerated, as in the printed figure.



The derivation of the equivalence whose scene this is can be opened step by step, each step with its premises, its rule and the result of the check:

The two forms place the sun at the same point when R' = R and r = e

GIVEN

H1 the deferent has the radius of the eccentric
$R' = R$

H2 the epicycle has the eccentricity as its radius
r = e

ARGUMENT

1 **the sun of the eccentric form and the sun of the epicyclic form are one point** *cases* ✓
S = S′

- **1.1** S lies on the eccentric *construction* ∘
  ES = R from constr. S
- **1.2** C lies on the deferent *construction* ∘
  OC = R′ from constr. C
- **1.3** OE is the eccentricity *construction* ∘
  OE = e from constr. E
- **1.4** S' lies on the epicycle *construction* ∘
  CS′ = r from constr. S′
- **1.5** **Case: off the apsidal line, OESC is a parallelogram** ✓
  when OES a triangle from 1.5.7
  - **1.5.1** **ES and OC are parallel** ✓
    ES ∥ OC from 1.5.1.2
    - **1.5.1.1** both angles are the mean anomaly κ̄, counted from the apsidal line *construction* ∘
      ∠AES = ∠A′OC from constr. A, constr. S, constr. A′, constr. C
    - **1.5.1.2** the apsidal line falls on ES and OC and makes equal angles with them on the same side *Elements I.28* ✓
      ES ∥ OC from 1.5.1.1
  - **1.5.2** ES = R = R' = OC *common notion 1* ✓
    ES = OC from 1.1, 1.2, H1
  - **1.5.3** **OE and CS are parallel: OESC is a parallelogram** ✓
    OE ∥ CS from 1.5.3.1
    - **1.5.3.1** the lines joining the ends of the equal and parallel ES and OC *Elements I.33* ✓
      OE ∥ CS from 1.5.1, 1.5.2
  - **1.5.4** and they are equal *Elements I.33* ✓
    OE = CS from 1.5.1, 1.5.2
  - **1.5.5** CS = OE = e = r = CS' *common notion 1* ✓
    CS = CS′ from 1.5.4, 1.3, H2, 1.4
  - **1.5.6** the radius of the epicycle keeps the direction of the apsidal line *construction* ∘
    CS′ ∥ OE from constr. S′, constr. E
  - **1.5.7** S and S' lie from C along the parallel to OE, the same way, at the same distance *lemma same ray* ✓
    S = S′ from 1.5.3, 1.5.6, 1.5.5
- **1.6** **Case: at the apogee the four points lie on the apsidal line, E between O and S** ✓
  when O, E, S on one line; E between O and S from 1.6.7
  - **1.6.1** OS = OE + ES *lemma segment addition* ✓
  - **1.6.2** OS' = OC + CS' *lemma segment addition* ✓
    OS′ = OC + CS′
  - **1.6.3** e + R = R' + r *algebra* ≈
    OS = OS′ from 1.6.1, 1.6.2, 1.1, 1.2, 1.3, 1.4, H1, H2
  - **1.6.4** S lies on the apsidal line *lemma one line* ✓
    OS ∥ OE from the case
  - **1.6.5** and so does S' *construction* ∘
    O, E, S′ on one line from constr. C, constr. S′, constr. E
  - **1.6.6** OS′ ∥ OE *lemma one line* ✓
    from 1.6.5
  - **1.6.7** S and S' lie from O along the apsidal line, the same way, at the same distance *lemma same ray* ✓
    S = S′ from 1.6.4, 1.6.6, 1.6.3
- **1.7** **Case: at the perigee the four points lie on the apsidal line, O between E and S** ✓
  when O, E, S on one line; O between E and S from 1.7.7
  - **1.7.1** ES = EO + OS *lemma segment addition* ✓
  - **1.7.2** CO = CS' + S'O *lemma segment addition* ✓
    CO = CS′ + S′O
  - **1.7.3** R − e = R' − r *algebra* ≈
    OS = OS′ from 1.7.1, 1.7.2, 1.1, 1.2, 1.3, 1.4, H1, H2
  - **1.7.4** S lies on the apsidal line *lemma one line* ✓
    OS ∥ OE from the case
  - **1.7.5** and so does S' *construction* ∘
    O, E, S′ on one line from constr. C, constr. S′, constr. E
  - **1.7.6** OS′ ∥ OE *lemma one line* ✓
    from 1.7.5
  - **1.7.7** S and S' lie from O along the apsidal line, the same way, at the same distance *lemma same ray* ✓
    S = S′ from 1.7.4, 1.7.6, 1.7.3

THEREFORE

∴ **in every state the two forms have the sun at one point** ✓
S = S′ from 1

3 The findings

Table 1 and Table 2 list the 45 assertions. Thirty-five are confirmed. One — that the distinction between the real and the apparent non-uniformity of the sun's motion "is recognized only by Kepler" — is a judgement about the history of ideas that no arithmetic reaches; it is listed as not checkable, and left to the reader. Nine differ from recomputation. We take them in three groups.

Table 1 Assertions 1 to 24, pages 17 to 22, from the tool's report. "p." is the printed page; the quotation is as printed (the transcription's LaTeX reduced; long quotations cut). Printed and recomputed values are sexagesimal; for a geometrical assertion "holds" means that the proposition holds over the model's parameter space, or that the step of the derivation is true and licensed by its rule. ✓ confirmed to the last printed place; ✕ a discrepancy, with its attribution (print: the author's own statement; rep.: a figure he reports from a source); — not checkable. The formalisation of every item is in the YAML file.

#	p.	assertion (quoted)	printed	recomputed	verdict
1	17	365 1/4 - 1/300 days = 365;14,48^d	365;14,48	365;14,48	✓
2	17	365;14,48^d = 365^d 5;55,12^h	5;55,12	5;55,12	✓
3	17	is 0;6,26^h too long	0;6,26	0;6,26	✓
4	17	3760/304 · 29;31,50,8,20^d ≈ 111,035^d/304	111035	111035	✓
5	17	111,035^d/304 = 365;14,48,9…^d	365;14,48,9	365;14,48,9	✓
6	17	365;15,25^d = 365^d 6;10^h	6;10	6;10	✓
7	17	1° per century or 36‘‘ per year	36	36	✓
8	19	on which the sun S moves uniformly about E …	holds	holds	✓
9	19	complete parallelogram OESC	holds	holds	✓
10	19	so that r is always parallel to the apsidal…	holds	holds	✓
11	20	the direction OS from the earth to the sun,…	holds	holds	✓
12	20	the apparent nonuniform motion of the sun i…	holds	holds	✓
13	20	in both models the directions OS and OS′ ar…	holds	holds	✓
14	20	There is, however, a distinction between th…	—	—	—
15	22	The mean anomaly κ̄ = λ̄s − λA	holds	holds	✓
16	22	the true anomaly κ = κ̄ ± c about O	holds	holds	✓
17	22	sin c = e sin κ̄/(R^2 + e^2 + 2Re cos κ̄)^1…	holds	holds	✓
18	22	tan c = e sin κ̄/R + e cos κ̄	holds	holds	✓
19	22	c < 0° for κ̄ > 180°, c > 0° for κ̄ > 180°	c < 0° for κ̄ > 180°, c > 0° for κ̄ >…	c > 0° for κ̄ < 180°, c < 0° for κ̄ >…	✕ print
20	22	lies on a line parallel to ES in the direct…	holds	holds	✓
21	22	vernal equinox to summer solstice 94½d with…	93;9	93;9	✓
22	22	summer solstice to autumnal equinox 92½d wi…	91;11	91;10	✕ rep.
23	22	a total of 187d and mean motion of 184;20°	184;20	184;19	✕ rep.

#	p.	assertion (quoted)	printed	recomputed	verdict
24	22	a total of 187d	187	187	✓

Table 2 Assertions 25 to 45, pages 22 and 23.

#	p.	assertion (quoted)	printed	recomputed	verdict
25	22	e = 2;29,30 ≈ 2;30, thus e/R = 1/24	0;2,30	0;2,30	✓
26	22	from which he finds that where R = 60, e = …	2;29,30	2;29,28	✕ rep.
27	22	and λA = 65;30°	65;30	65;35	✕ rep.
28	22	vernal equinox to summer solstice 94½d	2;29,30	2;28,58	✕ rep.
29	22	summer solstice to autumnal equinox 92½d	65;30	65;26	✕ rep.
30	22	the maximum correction cm = sin−1e/R = 2;23°	2;23	2;23	✓
31	22	e is in error by about +0;24	0;24	0;24	✓
32	22	+0;24/2;6 = +19%	19	19	✓
33	22	λA by −0;46° in −145	−0;46	−0;46	✓
34	22	−5;36° in 140	−5;36	−5;36	✓
35	22	its longitude has increased by +4;50° in 28…	4;50	4;50	✓
36	22	of which about 4° is the precession of the …	4	4	✓
37	22	0;50° the proper or sidereal motion of the …	0;50	0;50	✓
38	22	al-Battānī found e = 2;4,45 and cm = 1;59,1…	1;59,10	1;59,9	✕ rep.
39	22	λA = 82;17°, a notable advance of 16;37° in…	16;37	16;47	✕ print
40	23	λA = 77;50°, a regression of 4;27°	4;27	4;27	✓
41	23	cm = 2;10°, corresponding to e ≈ 2;16	2;16	2;16	✓
42	23	λs = λs + c = 180° + 2;10° = 182;10°	2;10	2;10	✓
43	23	κ̄ = 182;10° −65;30° = 116;40°	116;40	116;40	✓
44	23	λs ≈ 183;15°, so Ptolemy's mean longitude …	−1;5	−1;5	✓
45	23	the error increases by −0;26,25° per century	−0;26,25	−0;26,25	✓

3.1 Discrepancies in reported figures: roundings

Three discrepancies concern numbers that Swerdlow reports from his sources, and all three are of the same kind: the source's rounding.

Ptolemy's arcs of mean motion (page 22, items 22 and 23). Ptolemy's intervals from vernal equinox to summer solstice and from summer solstice to autumnal equinox are 94½ and 92½ days, "with mean motion 93;9°" and "91;11°", "and a total of 187^{d} and mean motion of 184;20°". With Ptolemy's own year of 365;14,48 days the mean motion is 0;59,8,17,…° per day, and exact arithmetic gives 93;8,33°, 91;10,17° and 184;18,50°. The first rounds to 93;9° as printed; the second rounds to 91;10°, not 91;11°; and 184;20° is the sum of the two rounded arcs, not the rounding of the exact total. These are Ptolemy's figures, and Swerdlow reports them; the discrepancy is between Ptolemy's rounding and exact arithmetic. It is small — 43 seconds of arc, and 1;10′ — but a reader who recomputes the page will meet it. (We have not traced how Ptolemy obtained 91;11°.)

Al-Battānī's maximum equation (page 22, item 38). Regiomontanus reports that al-Battānī found e = 2;4,45 and c_m = 1;59,10°. The maximum equation of the eccentric model is arcsin(e/R); for e = 2;4,45 and R = 60 that is 1;59,9°, one second short of the reported 1;59,10°. Al-Battānī's c_m is his own value as Regiomontanus transmits it, and we have not checked it against al-Battānī's *Zīj*; one second is within the roundings of a sexagesimal computation of the sine. (The first version of this paper gave 1;59,8°: the figure had been truncated where it should have been rounded.)

3.2 Discrepancies in reported figures: the parameters from the seasons

Four discrepancies belong to one sentence. Having given the two seasons, the page continues: "from which he finds that where R = 60, e = 2;29,30 ≈ 2;30 … and λ_A = 65;30°" (Figure 2). This is an assertion that something follows, and the model lets it be tested. The eccentric model has two unknown parameters here, the eccentricity and the longitude of the apogee, and the two intervals are two conditions on them. The tool inverts the model numerically, twice (items 26 to 29).

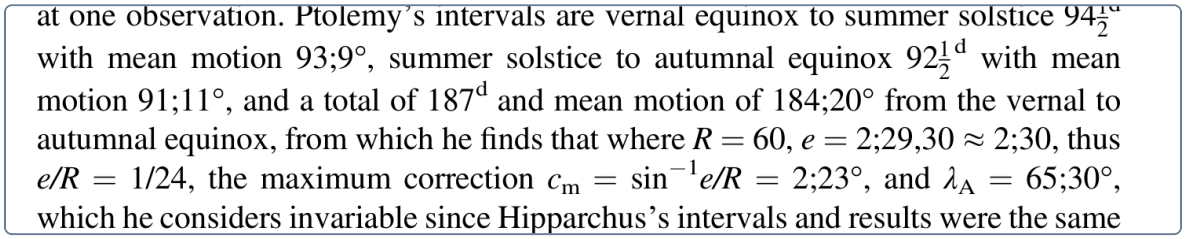

at one observation. Ptolemy's intervals are vernal equinox to summer solstice $94\frac{1}{2}^{d}$ with mean motion 93;9°, summer solstice to autumnal equinox $92\frac{1}{2}^{d}$ with mean motion 91;11°, and a total of 187^{d} and mean motion of 184;20° from the vernal to autumnal equinox, from which he finds that where $R = 60$, $e = 2;29,30 \approx 2;30$, thus $e/R = 1/24$, the maximum correction $c_m = \sin^{-1} e/R = 2;23°$, and $\lambda_A = 65;30°$, which he considers invariable since Hipparchus's intervals and results were the same

Figure 2 Page 22, scan page 52, `p05`: the seasons and the parameters Ptolemy finds from them.

Table 3 Eccentricity and apogee determined through the model. In both cases the solution found is the only one.

inverted for	e	λ_A
the arcs of mean motion as reported, 93;9° and 91;11°	2;29,28	65;35°
the day counts, 94½ and 92½ days, with the year 365;14,48 d	2;28,58	65;26°
reported	2;29,30	65;30°

From the two reported arcs the model gives Ptolemy's eccentricity to two units of the second sexagesimal place and his apogee to 0;5°. From the day counts, with exact arithmetic, it gives an eccentricity half a minute smaller and an apogee 0;4° lower. Ptolemy's parameters follow from his rounded arcs, to within what a computation with a table of chords loses, and not from the day counts. The two cases differ only in the rounding of the arcs, by 27 and by 43 seconds; that moves the apogee by 0;9° and the eccentricity by 0;0,30. Swerdlow says of this derivation that it "is so sensitive to errors in the intervals of time … that the exact results are fortuitous"; the two determinations show that sensitivity inside Ptolemy's own numbers. Ptolemy's steps are not on the page, and we have not compared with the *Almagest*; these four discrepancies are facts about the reported derivation, not about Swerdlow's page.

3.3 Misprints

Two discrepancies concern the page itself. In both cases the arithmetic of the sentence does not come out, and the page image confirms the transcription.

The sign rule (page 22, item 19). After giving the two formulae for the equation of centre c, the page states when it is to be added and when subtracted:

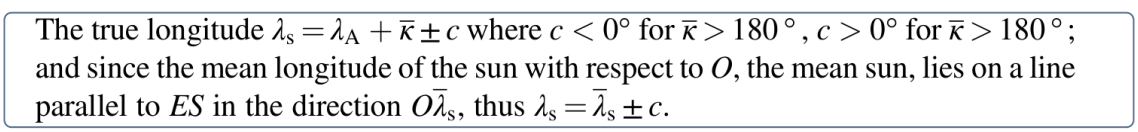

The true longitude $\lambda_s = \lambda_A + \overline{\kappa} \pm c$ where $c < 0°$ for $\overline{\kappa} > 180°$, $c > 0°$ for $\overline{\kappa} > 180°$; and since the mean longitude of the sun with respect to O, the mean sun, lies on a line parallel to ES in the direction $O\overline{\lambda}_s$, thus $\lambda_s = \overline{\lambda}_s \pm c$.

Figure 3 Page 22, scan page 52, `p04`: the sign rule as printed.

> "The true longitude $\lambda_s = \lambda_A + \bar{\kappa} \pm c$ where $c < 0°$ for $\bar{\kappa} > 180°$, $c > 0°$ for $\bar{\kappa} > 180°$".

Both clauses name the same condition, $\bar{\kappa} > 180°$; as printed the rule is contradictory and says nothing about $\bar{\kappa} < 180°$. The model settles the matter. In the eccentric model under Ptolemy's parametrisation (`fr:astra/solar-eccentric` with `fr:astra/solar-eccentric.ptolemy`), c as the difference between the mean and the true longitude is positive in the first half of the anomaly and negative in the second: the tool reads off the rule "$c > 0°$ for $\bar{\kappa} < 180°$, $c < 0°$ for $\bar{\kappa} > 180°$". The printed tangent formula, $\tan c = e \sin \bar{\kappa} / (R + e \cos \bar{\kappa})$, gives the same sign (its numerator changes sign at 180°), and the derivation of that formula from the geometry holds in absolute value over the whole parameter space. One of the two printed clauses must therefore read $\bar{\kappa} < 180°$. Which one depends on the convention for the sign of c, and the formula fixes it: $c > 0°$ for $\bar{\kappa} < 180°$, and $\lambda_s = \lambda_A + \bar{\kappa} - c$ there, since the true longitude lags behind the mean on the way from apogee to perigee. The misprint is a dropped inequality sign. It matters because the sentence is the one place on the page where the reader is told how to apply c.

Al-Battānī's apogee (page 22, item 39). The last sentence of the page reads:

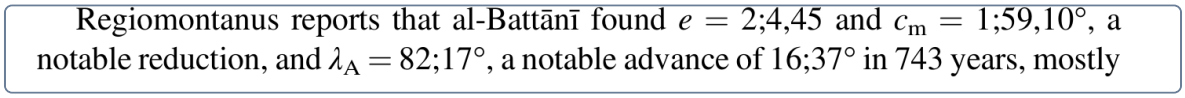

Regiomontanus reports that al-Battānī found $e = 2;4,45$ and $c_m = 1;59,10°$, a notable reduction, and $\lambda_A = 82;17°$, a notable advance of 16;37° in 743 years, mostly

Figure 4 Page 22, scan page 52, `p06`: al-Battānī's values as printed.

> "Regiomontanus reports that al-Battānī found $e = 2;4,45$ and $c_m = 1;59,10°$, a notable reduction, and $\lambda_A = 82;17°$, a notable advance of 16;37° in 743 years, mostly from precession".

Ptolemy's apogee, on the same page, is $\lambda_A = 65;30°$. The advance from 65;30° to 82;17° is 16;47°, not 16;37°. The difference is exactly 0;10°, a single sexagesimal digit, and no other base on the page yields 16;37°: al-Battānī's own apogee is given relative to Ptolemy's, the 743 years run from Ptolemy's epoch to al-Battānī's, and the next sentence computes az-Zarqāl's "regression of 4;27°" from the same 82;17° correctly (82;17 − 77;50 = 4;27, item 40). The 16;37° is a misprint for 16;47°. The comparison of the two parametrisations of the model, Ptolemy's and al-Battānī's, shows the same difference, +16;47°. It is harmless to the argument — "mostly from precession" holds for either — and it would pass any reader who does not subtract.

3.4 What was confirmed

The confirmations deserve a word, because they are the bulk of the result and because some of them are not trivial. The arithmetic of the year holds throughout: 365¼ − 1/300 days is 365;14,48 days, which is 365 days 5;55,12 hours, which is 0;6,26 hours longer than the modern mean tropical year of 365.2422 days; Hipparchus's Metonic derivation, 3760 synodic months of 29;31,50,8,20 days over 304 years, gives 111,035 days and 365;14,48,9 days; the sidereal year of 365;15,25 days is 365 days 6;10 hours; 1° per century is 36″ per year. Ptolemy's parameters hold: e/R = 2;30/60 = 1/24, the maximum equation $\arcsin(1/24) = 2;23°$. Swerdlow's error analysis of page 22 holds to the last digit: against the "correct" values $e = 2;6,22$ and $\lambda_A = 66;16°$ for −145 and $e = 2;5,37$ and $\lambda_A = 71;6°$ for 140, Ptolemy's e is in error by +0;24 or +19 %, his λ_A by −0;46° and −5;36°, and the apogee has advanced 4;50° in 285 years, of which about 4° (285 years of precession at the modern rate) is precession and 0;50° proper motion. On page 23, the Alfonsine maximum equation 2;10° corresponds to $e \approx 2;16$; Ptolemy's mean longitude at the autumnal equinox is 180° + 2;10° = 182;10°, and the model under Ptolemy's values, solved for the equation of centre at true longitude 180°, indeed gives 2;10°; the mean anomaly 182;10° − 65;30° = 116;40°; the error of −1;5° against the correct 183;15°; and the growth of that error by −0;26,25° per century, which follows from the excess of 0;6,26 hours in the year turned into mean motion.

The geometrical assertions hold, each as what it is. Properties of the constructions hold as propositions of the two models, for every parameter value: the sun moves uniformly about E through $\bar{\kappa}$ from the apogee; the radius of the epicycle stays parallel to the apsidal line; $\bar{\kappa} = \bar{\lambda}_s - \lambda_A$; $\kappa = \bar{\kappa} \pm c$. The statements of the argument hold as steps of derivations: OESC is a parallelogram (Elements I.33, from ES and OC equal and parallel); the two suns coincide, so that the direction OS is the same in both forms; under $R'/r = R/e$ the directions OS and OS′ are the same (Elements VI.6 and I.29). Both printed formulae for c, the tangent form and the sine form, are conclusions of a derivation in the eccentric model.

3.5 What the derivations show beyond the assertions

Writing the argument of page 19 as a derivation, with a rule for every step, shows two things that are not assertions of the page and therefore not items of the table.

The equivalence has hypotheses. "Complete parallelogram OESC and let C be the center of an epicycle of radius r = e … on a concentric circle about O of radius R′ = R" gives them in passing. In the derivation they are the two givens. Lifting either makes the conclusion fail, and the check that no step uses a given it does not cite finds none used tacitly.

The parallelogram argument does not cover the apsides. Elements I.33 needs the two equal and parallel lines not to lie in one line, and I.28, by which ES and OC are parallel, needs the two points on one side of the transversal. At $\bar{\kappa} = 0°$ and $\bar{\kappa} = 180°$ the points O, E, S and C lie on the apsidal line (Figure 1), and the two propositions have nothing to apply to. The conclusion is true there as well — the sun of both forms is at the apogee or at the perigee — but the argument by the parallelogram does not reach it. The derivation therefore has three cases: off the apsidal line by the parallelogram, and at the apogee and at the perigee by adding and subtracting lengths along the line (OS = e + R and OS′ = R′ + r; R − e and R′ − r). It is displayed in full, with every step, in [3]. A figure drawn at $\bar{\kappa} \approx 41°$ shows the first case only. This is not an error of the page; it is what the page's one sentence of argument leaves to the reader.

4 What the reading adds

A careful reader could have found the two misprints with a pencil. Three things distinguish the deep reading from that reader.

Coverage. The reader checks what catches the eye; the reading checks every assertion, because every assertion is an item, and an item without a formalisation has to say why. The tool runs the 45 items in seconds. Nothing was skipped as too obvious — and the sign rule, which one reads past because one knows what it means, is where the misprint was.

Attribution. Each discrepancy is classified at once: is this Swerdlow's arithmetic or Ptolemy's? The seven discrepancies in reported figures would otherwise look like errors of the page; they are facts about Ptolemy's computation and about Regiomontanus's report of al-Battānī, and they are worth knowing as such.

Each assertion as what it is. A page of this kind asserts things of four kinds: that a construction has a property, that something follows, that a number results from other numbers, that parameters result from observations. They are checked differently. A property is checked for every parameter value, not only for the drawn one. A step is checked for truth and for whether the proposition that would license it has its hypotheses and its configuration. A number is recomputed. A determination is inverted through the model. The sentence on the parameters from the seasons would pass a reader who checks arithmetic only: there is no arithmetic on the page to check. It is an assertion that something follows, and it can be tested only with the model.

And one thing distinguishes the reading from a footnote: it is a record that runs. The YAML file is the apparatus; the report is reproducible; every item names its paragraph, and the paragraph is one address away. A later reader who disagrees changes an item and runs it again.

5 Limits

The reading covers five pages, not the chapter, and it covers what can be formalised: arithmetic, geometry, and the consistency of the page with itself. It does not check Swerdlow's historical claims against the sources he cites — that Ptolemy's intervals are those of the *Almagest*, that al-Battānī's values are as Regiomontanus reports them — except where the arithmetic on the page or the model makes a claim testable. The checks of the derivations test truth on samples of the parameter space and match rules as schemas; they are not formal proofs [2]. It takes the modern values Swerdlow uses (the mean tropical year, the rate of precession) as given where he does not state them, and says so in the item's note.

The transcription is the OCR of the book's PDF with its mathematics in LaTeX; the tool normalises that markup before matching a quotation, and two quotations had to be written in the transcription's flattened form of a fraction. The evidence crops are cut by coordinates read off a grid over the page image, not by OCR line boxes, which the page XML does not have.

Finally, "misprint" is a judgement. We call the two cases misprints because the arithmetic of the sentence does not come out as printed, because the page image confirms the transcription, and because a one-character change makes each sentence correct. We have not seen Swerdlow's manuscript; the book was published after his death, and where the slip entered is not ours to say.

6 Conclusion

Five pages of a learned book, read deeply, yield 45 assertions. Thirty-five hold and one is beyond arithmetic. Three show the roundings of Ptolemy and al-Battānī. Four show that Ptolemy's eccentricity and apogee follow from his rounded arcs and not from the day counts of the seasons. Two are misprints: a dropped inequality sign in the rule for the equation of centre, and 16;37° for 16;47° in the advance of al-Battānī's apogee. The derivation behind the page's parallelogram, written out, needs two further cases at the apsides. None of this changes what the pages say about the history of astronomy, and all of it changes how far a reader can trust them without checking. The reading is kept as a file that runs, with an address for every assertion; that, rather than the two misprints, is the result we would like to see become ordinary.

Availability

The reading is `readings/swerdlow-solar.yaml` in the Famulus workspace `famulus-astronomica`, run by `famulus-astra reading`; its report and the evidence crops are in `build/readings/swerdlow-solar/`. The astra/1 texts — two models, their parametrisations, the superposition and four derivations — and the language are described in the companion paper [2]. Swerdlow's book is cited by its scan pages in the Famulus collection `kepler-sec` (scan page n is printed page n − 30 in chapter 1).

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[2] G. Graßhoff, "astra/1. A Language for Historical Astronomical Models and the Geometrical Reasoning about Them," 2026.

[3] G. Graßhoff, "Two Geometrical Arguments Reconstructed. Euler on the Tide-Raising Force, and the Equivalence of Eccentric and Epicycle," 2026.